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Deriving Additional Parameters from the Staple Force vs Extension Curve Measured by ATLAS

By

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SUMMARY

This report outlines the increasing complexity of the tensile stress vs strain curves as we progress from a metal, to an individual wool fibre and, finally, to a wool staple. In each case parameters can be obtained from these curves to quantify aspects of the stress vs strain relationships for these materials, e.g., Young's Modulus for a metal or an individual wool fibre and Decrimped Staple Length for a wool staple.

A procedure is described in which the PC-ATLAS software was modified for research purposes to capture the Strength (force) vs Extension curve for a staple during a standard ATLAS test. The curve for a staple shows a region of crimp take-up, a region of rapid ascent, a point of apparent yield, a region of gradual ascent, a peak, a descent, and a tail. The principles behind using this curve to calculate the Decrimped Staple Length, Specific Work of Rupture of Staple, and other parameters are discussed.

Decrimped Staple Length is the only measurement being evaluated for possible commercial use at this point in time.

BACKGROUND

A new ATLAS measurement, Decrimped Staple Length (DSL), was presented to the last IWTO conference [1]. The report briefly touched on the theory behind the Stress-Strain curves measured by ATLAS, and acknowledged the development of a mathematical model.

This paper presents a more extensive description of the model, developed by the author, which was the basis of this measurement. The material draws broadly from Morton & Hearle [2] and raises concepts covered by Yu [3] in the context of greasy wool.

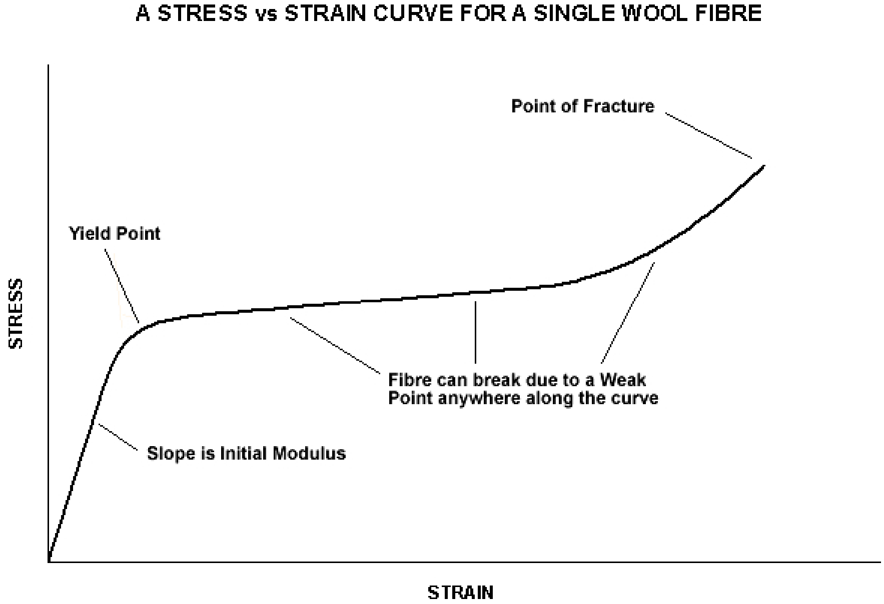
The Stress vs Strain Curve of a Single Wool Fibre

The basic physics that describe the stress vs strain curves for metals are summarised in Appendix 1. The Appendix also describes a simplified stress-strain theory for fibres and a method for calculating the total Energy used (or Work Done) to break a staple of wool.

A schematic of a typical stress vs strain curve for a single straight fibre is shown in Figure 1.

The tensile properties of wool fibres only resemble those of metal specimens in the initial phase of extension (or strain). Extension is assumed to commence when the fibre is straight (i.e. all crimp has been taken up by the apparatus). The term "Initial Modulus" is used to describe the initial slope of the Stress vs Strain relationship. (Here we revert to the use of Specific Stress as applied in Textile Physics, hence Initial Modulus has the units of Specific Stress).

Figure 1



The Yield Point is identified in Textile Physics, though there are various geometric definitions of its exact position [4]. This point is followed by a flat ascent (sometimes known as a "Yield Region"), eventually switching to a rapid ascent (or "Post-Yield Region"), followed by fracture.

If the fibre breaks at the end of the Stress vs Strain curve, the break will occur at the thinnest point existing at the moment of fracture unless the fibre has a structurally weak point, which is not necessarily at the thinnest point. Weak-point breakage can occur anywhere along the curve.

The Stress vs Strain Curve of a Wool Staple

Figure 2

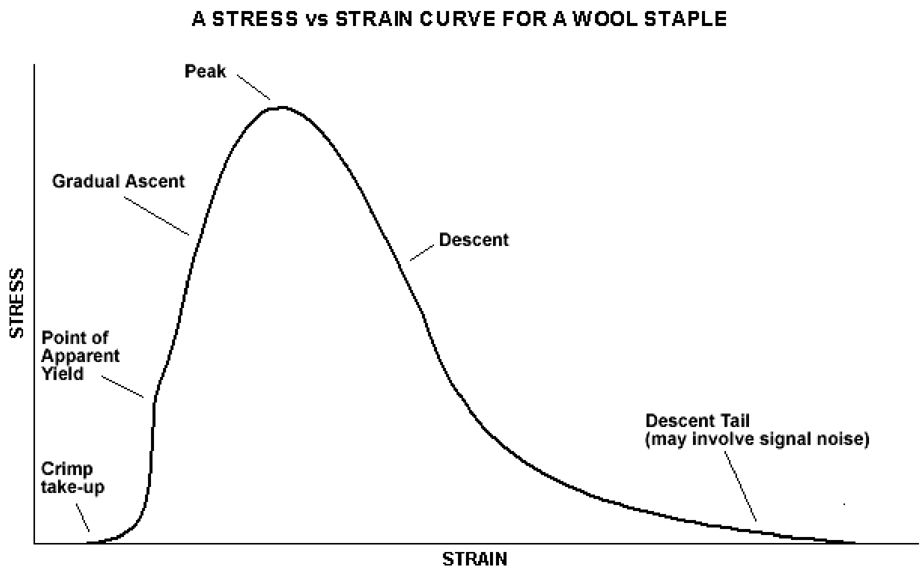


Figure 2 is based upon Force vs Extension results for a real staple, which is a bundle of wool fibres, and is a typical outcome for ATLAS testing. The schematic has been smoothed to clearly illustrate six identifiable points, or zones, of interest:

- The initial stage is marked by crimp take-up. Work done in this region can be considered as the Energy of Crimp Take-up, and does not contribute to the Work of Rupture of the staple.

- When the crimp is taken up, extension commences within an elastic deformation region for a fibre. The slope is the Initial Modulus of the staple.

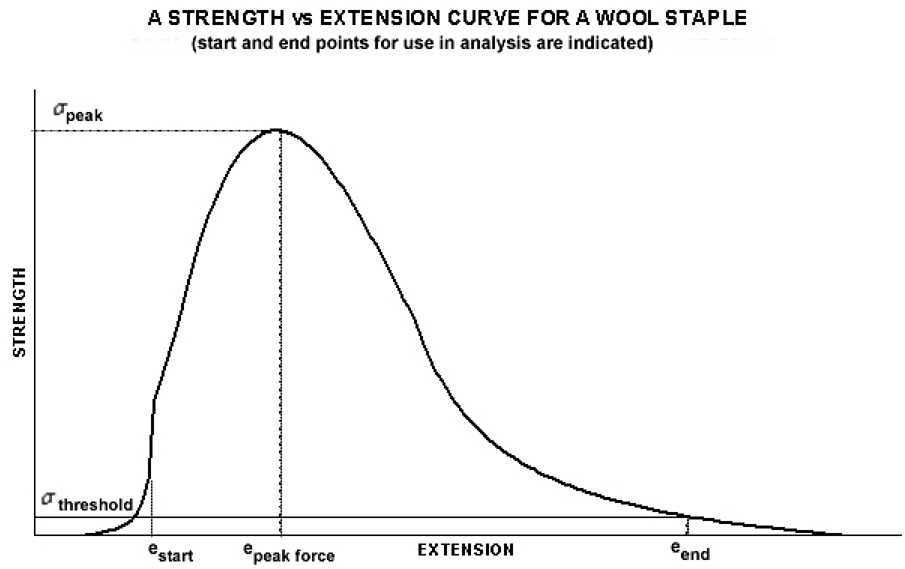
  There is typically a change of slope, or Point of Apparent Yield, marking the end of the region of elastic deformation of individual fibres.

- The curve exhibits a slowing ascent as individual fibres start to break.
- A point of peak stress is reached.
- A region of descent appears as fibres continue to break.
- Finally a right hand tail is reached. The noise in this region may be due to the ATLAS, and/or to broken fibres sliding over each other.

The peak Strength is the Staple Measurement that is used by ATLAS to calculate the Mean Staple Strength of greasy staples. This measurement does not purport to represent the intrinsic strength of the wool keratin that makes up the fibres. The ATLAS Staple Strength is more a reflection of the minimum cross sectional area of the fibres within the bundle and between the jaws of the instrument.

Clearly, from Figure 2, it is evident that in order to derive any new measurements from the stress vs strain curve, “start” and “end” points must be defined (see Figure 3). Extension can be converted to fractional Strain by dividing it by a de-crimped initial length between the ATLAS jaws (i.e. the "Decrimped Staple Length"). "Specific Stress" and "Strength" are terms that can be used interchangeably to label an axis. Furthermore it is more convenient to use the term “strength” or “force” as substitute for “stress”; and “extension” as a substitute for “strain”, as these are the parameters actually recorded by the ATLAS instrument. For any staple, these changes do not affect the shape of the curve shown in Figures 2 and 3.

Figure 3



DERIVATION OF THE NEW MEASUREMENTS

Source Data

Greasy staple measurements using ATLAS usually fall under a very different testing regime to other fibre strength measurements. The Staple Length is variable and the Fibre Length within the staple is unknown. The ATLAS speed of extension is very rapid (0.3 $m.s^{-1}$) since the ATLAS test was designed to provide and indictor of fibre breakage during processing, a process involving rapid rates of extension.

The PC-ATLAS software was modified to save the Force vs Time data of staples being broken. ATLAS collects a force transducer reading at every clock tick (i.e., the time interval defined by the processor of the ATLAS computer) from when the moving carriage is instructed to break the staple, up to 153 readings including the start point. Each clock tick marks 1/300th of a second. The force transducer readings are in ADC units, and are converted to Newtons.

Measurements of the velocity of the ATLAS moving carriage were undertaken prior to the Decrimped Staple Length trial [1, 7]. These showed that the acceleration of the moving carriage increased smoothly from 0 $m.s^{-2}$ to 5 $m.s^{-2}$ over 0.05 seconds; then acceleration decreased smoothly back to 0 $m.s^{-2}$ over 0.06 seconds. These accelerations took the moving carriage from 0 $m.s^{-1}$ to 0.3 $m.s^{-1}$.

The data recorded by the modified ATLAS software for each staple includes:

Staple ID Number

Staple Length, L (mm)

Peak Staple Force, F (N)

Tip Mass, TM (mg)

Base Mass, BM (mg)

Force measurements, F_i (N) for each clock tick i.

While the extension data is recorded as clock ticks it is a relatively trivial exercise to convert these to linear distance or extension. In doing so allowance has to be made for the fact that there is an initial acceleration phase until the moving jaw of the ATLAS attains constant velocity.

Definition of “start’ and “end” points

In order to obtain parameters to characterise the Strength vs Extension curves it was necessary to define the “start” and “end” points highlighted in Figure 3.

There are two options for defining the “start” point:

1. the intersection of the extrapolation of the slope of the Initial Modulus and the Extension (or strain) axis (Figure 4); or,
2. the point on the Extension (or strain) axis where 2.5% of the total Specific Work of Rupture has been used (Figure 5).

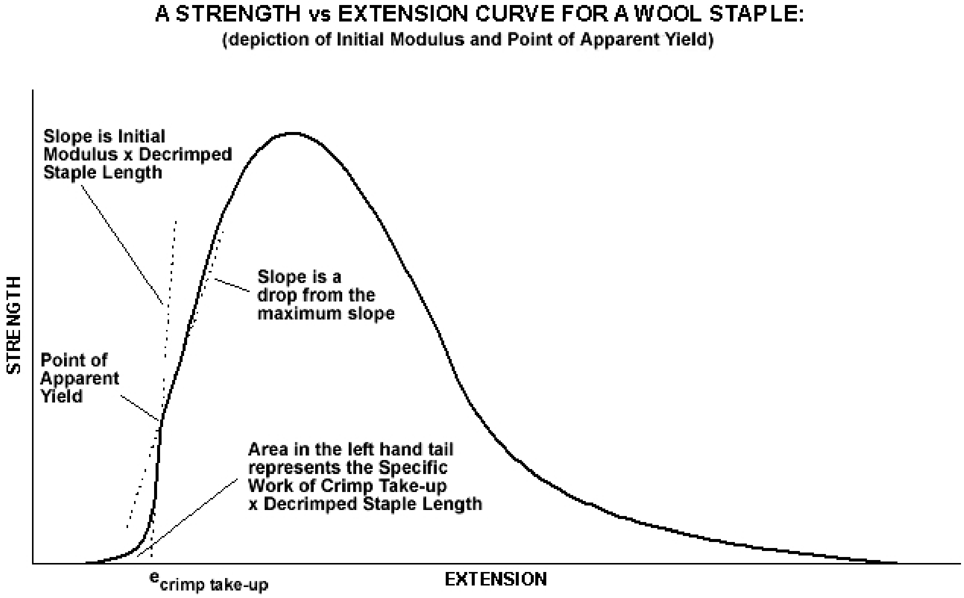
Similarly, there are two options to define the “end” point:

1. specify a strength threshold which exceeds the noise level of the signal (Figure 6); or
2. the point on the Extension (or strain) axis where 97.5% of the total Specific Work of Rupture has been used (Figure 5).

Each of these options has been included in the software for evaluation of the most appropriate “start” and “end” points for the definition of a series of parameters discussed in the next section.

Decrimped Staple Length

Figure 4



There is a section of the Strength vs Extension curve after the crimp take-up where the slope reaches a maximum (Figure 4). The product of that slope and the Decrimped Staple Length is the best estimate of the Initial Modulus that the Strength vs Extension curve can provide. It represents the performance of the staple under load before any fibres reach their Yield Point or commence breaking.

For a single fibre this line of maximum slope can be extrapolated to the Strength = 0 axis [6]. The intercept is the Extension of Crimp Take-up ($e_{crimp\ take\text{-}up}$). It is possible to apply the same analysis to the Strength vs Extension curve of a staple. Hence the Decrimped Staple Length can be calculated from the unextended Staple Length (SL):

$$Decrimped\,StapleLength = \frac{(SL - JawAllowance + Extension\ at\,"start")xSL}{(SL - JawAllowance)}$$

where, *Jaw Allowance* = Length of staple covered by the jaws, and

Extension at "start" = either $e_{crimp\ take\text{-}up}$ or $e_{.025}$ (Figure 5).

Yield Point

The typical Strength vs Extension curve for staples tested by ATLAS has a discernable change of slope at the end of the section of maximum slope. This is probably due to the bulk of fibres leaving the region of elastic deformation.

This point, known as the Yield Point, can be determined as the intersection of the (Initial Modulus x Decrimped Staple Length) slope and the slope of the region immediately following a change in the Initial Modulus (Figure 4).

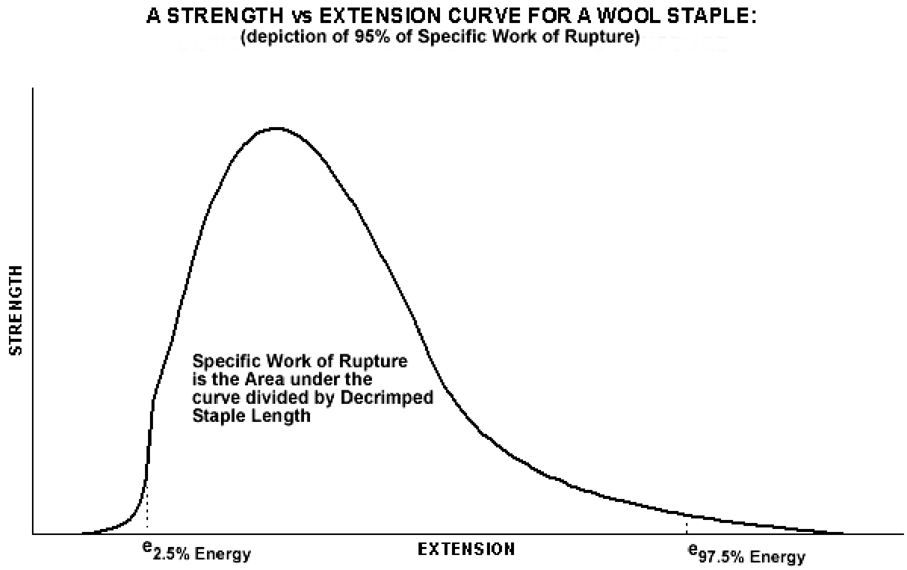
Specific Work of Rupture

In the early days of development of Staple Length & Strength measurements, CSIRO examined the use of energy to rupture staples as a predictor of Hauteur[9,10]. This was found to have no advantage over peak force. However, using the data provided by the new software it is possible to derive this information. This parameter provides a further illustration of the two options for defining the "start" and "end" points for the integration of the area under the curve.

Firstly, the calculation of Specific Work Of Rupture can arbitrarily exclude the area to the left of the $e_{crimp\ take\text{-}up}$, i.e. the "start", and the area to the right of where the Specific Stress has fallen below 4 N/ktex

(Figure 6), the "end". The former is deemed Work Of Crimp Uptake, and the latter is considered a noise effect.

Figure 5

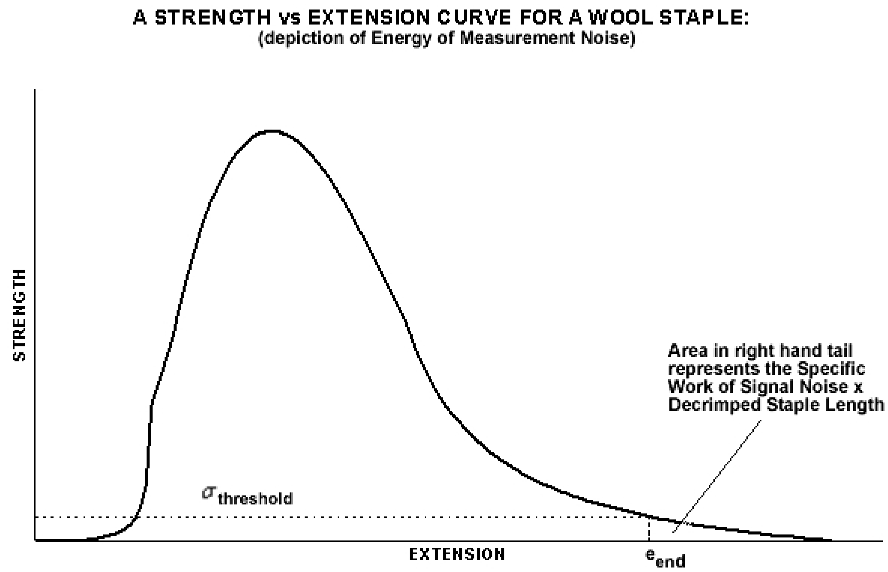


The second option considers a fixed portion of the Specific Work of Rupture, 95% (Figure 5). The area under the Strength vs Extension curve between the "start" and "end" is 95%, and the area of each excluded tail is 2.5%. With this approach:

$$\text{Average Specific Strength} = \frac{\int_0^\infty S.de}{e_{.975} - e_{.025}}$$

where $e_{.025}$ and $e_{.975}$ are the extensions at which 2.5 % and 97.5 % of the Specific Work of Rupture has been completed, respectively. The "95% Energy Analysis" is a simplified approach that does not assess an Initial Modulus or a Yield Strength. It would have an advantage in dealing with noisy or erratic force signals, and may have a place as a means of checking for gross errors.

Figure 6



CALCULATION METHODS

Details of the specific calculation methods incorporated into the software are provided in Appendix 2.

CONCLUSION

The ATLAS software has been modified for research purposes to provide some additional information about the staples being measured:

- Decrimped Staple Length (mm)
- Decrimping Ratio[1]
- Staple Specific Work of Rupture (N.ktex^{-1})
- Staple Initial Modulus (N.ktex^{-1})
- Staple Yield Strength (N.ktex^{-1})

This Staple Extension software is consistent with the current ATLAS instrument. It uses the same corrections to go from Peak Load to Staple Strength, but in this instance, records the whole Strength vs Extension curve. Recording the Strength vs Extension curve permits calculation of the intrinsic mechanical properties of staples using the five parameters listed above.

Research is required to ascertain whether any of these new parameters provide useful information for the specification of Greasy Wool. The most promising parameter under investigation is Decrimped Staple Length and its associated "Decrimping Ratio" [1] (which is the Decrimped Staple Length divided by the Staple Length).

Decrimped Staple Length is currently being investigated as a means of characterising the greater-than-normal extensibility of “Stretchy Belly” wool. This issue has been raised by Wool Exporters as a factor which may influence the processing performance of these wools.

ACKNOWLEDGEMENTS

The initial studies of the kinematics of the moving jaw carriage [7] were arranged by Feiming Liu and Chris Ruberg, with the analysis and interpretation of the data carried out by Victoria Fish, Joy Dempsey and Trevor Mahar (all current or former staff of AWTA Ltd’s Research and Development Division).

The Author also wishes to thank Joy Dempsey and Vicky Fish for their assistance in the evaluation of the commercial utility of these new measurements.

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APPENDIX 1

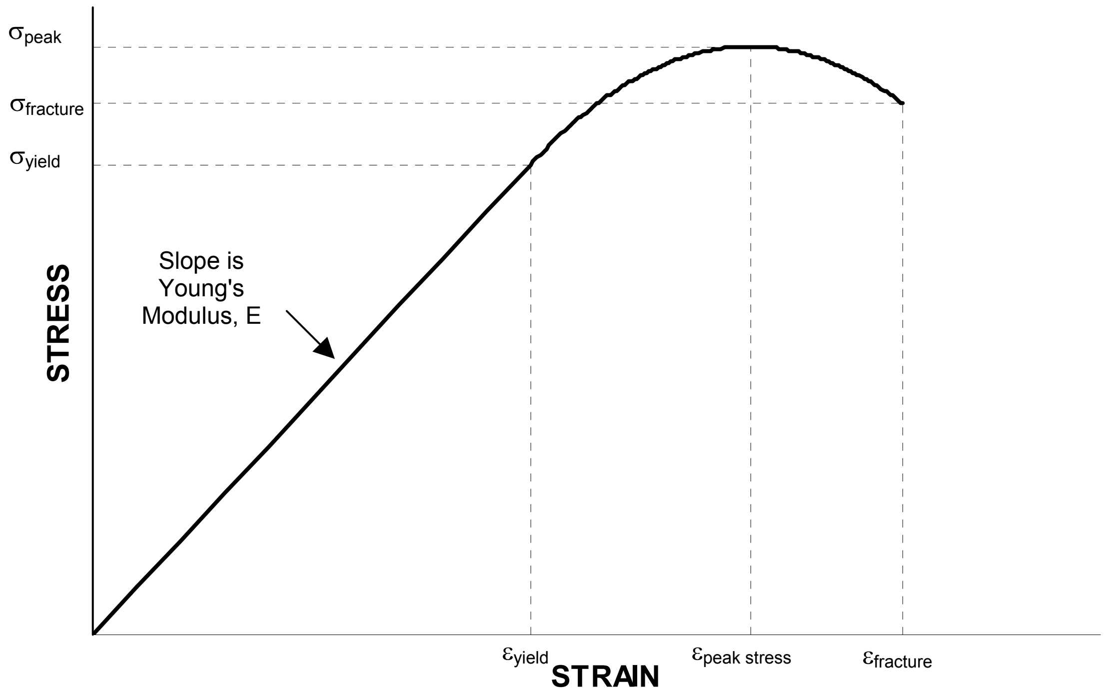
AN INTRODUCTION TO STRESS VS STRAIN CURVES

Tensile tests on soft metal test strips produce Stress vs Strain curves with a classic shape (Figure 1a). The specimen initially moves through a region of elastic deformation where the Stress vs Strain relationship is linear. The slope in this region is known as Young's Modulus (E), and it is used to calculate how the metal will perform under load. Its units are those of Stress. When the Strain reaches the Yield Point the sample enters a region of plastic deformation. The work done is now irreversible and the Stress vs Strain relationship becomes non-linear. The Stress will reach a peak and begin to decline, eventually reaching a Fracture Stress.

This Stress vs Strain relationship assumes that there are no flaws or secondary mechanisms (e.g. creep) in operation.

Figure 1a

A STRESS vs STRAIN CURVE FOR METALS



SIMPLIFIED STRESS-STRAIN THEORY FOR FIBRES

For a cylindrical straight, solid fibre (under no force) of Mass M, Length L, Diameter d, Density ρ and Cross-sectional Area A, the following relationships apply:

$M = \pi.(\ ½\ d\ )^2.L.\ \rho$

$A = \pi.(\ ½\ d\ )^2$

If the fibre is extended by Extension e to Length (L + e) under a tensile force F, the following hold:

Stress = F / A

Strain, ε = e / L

"Stress" and "Strength" can be used interchangeably if considered as scales. "Strength" is normally used as a description of Stress at a particular point in the extension process, eg peak load, fracture or yield.

This formula for Stress is strictly Nominal Stress. Assuming constant Density, conservation of Mass dictates that the Cross-sectional Area reduces as the fibre is extended. If it is assumed that the Cross-sectional Area does not vary along the length of the fibre, a "True Stress" could be calculated accounting for the reduction in Cross-sectional Area.

Textile applications use the fibre Linear Density as a basis to express Stress, i.e.

Specific Stress, σ = F x L / M

Specific Stress may also be called in various contexts Stress, Strength, Specific Strength or Tenacity. Some texts use S for the Specific Stress of individual specimens, and σ for the mean. The relationship between Stress and Specific Stress is proportional, i.e.:

Stress = $\rho.\sigma$

The energy required to break the fibre is:

Work of Rupture = $\int_0^\infty \mathrm{F.de}$

Specific Work of Rupture = Work of Rupture / M

= Work of Rupture / (Linear Density x L)

Here the word "specific" has implied division by mass rather than density.

An Average Strength may also be evaluated if suitable start and end points are defined:

$$\text{Average Strength} = \frac{\int_{start}^{end} \sigma.\mathrm{de}}{e_{end} - e_{start}}$$

CALCULATING TOTAL ENERGY

The total Energy used (or Work Done) to break a staple of wool can be calculated if the Force vs Extension curve is known:

Work of Rupture = $\int_0^\infty \mathrm{F.de}$

where F is Force (N) and e is extension (mm).

In order to compare results, the Strength vs Extension curve can provide meaningful values:

$$\text{Specific Work of Rupture} = \frac{\int_0^\infty \mathrm{F.de}}{M} = \frac{\int_0^\infty \mathrm{S.de}}{L}$$

where S is the Specific Strength (N/ktex). Real-world complications include consideration of the portion of the staple held in the jaws (i.e. its length and linear density), the effective mass upon which the force is exerted, and crimp take-up.

In practice, there may be valid reasons to limit the domain used to perform this integration. It may be desirable to exclude the Specific Work of Crimp Take-up. Signal noise in the right hand tail of the curve may also be excluded. These effects are trivial in comparison to the Specific Work of Rupture. The numerator can be divided by an appropriate domain (eg e_{start} to e_{end}) to produce an Average Specific Strength for the staple.

APPENDIX 2

CALCULATIONS

Measurement of Staple Strength by ATLAS uses the following calculations (refer IWTO-30):

Staple Mass (mg), SM = TM + BM

Correction Factor (dimensionless) for lost material during the staple breaking process,
STR_CF = 1.36704 - 0.006936 WB + 0.06126 FLC_CAT

Correction Factor (%) for the staple yield (i.e. clean wool content),
SY_CF = 0.83 WB + 0.314 VMB - 6.18 FLC_CAT + 29.6

Strength (N/ktex), S = 100 (L x F) / (SM x STR_CF x SY_CF)

For individual staples, the tensile load on the staple F_i (N) at extension e_i (mm) was measured at each ***i***th clock tick (***i*** = 0, 1, 2, ... n).

The author produced a set of example calculations so that the usefulness of three different approaches to using the Force vs Extension data for calculating Decrimped Staple Length and other parameters could be considered. These are described below.

The Strength (N) (i.e. Specific Stress) of the staple at the ***i***th clock tick,
S_i = 100 L x F_i / (SM x STR_CF x SY_CF). (Note that the Jaw Adjustment is ignored here).

Integration of the Strength vs Extension Curves

Rather than fitting curves to the data, discrete interval analyses are used.

The integral of the Strength vs Extension curve can be represented as the sum of thin vertical strips under the curve. These strips, or Differential Areas (DA_i's), are the product of the Applicable Decrimped Length (mm) and the Specific Work done on the staple ($N.ktex^{-1}$) between clock ticks (***i*** – 1) and ***i***.

The area of each strip, $DA_i = ½ (e_i - e_{i-1}) (S_i + S_{i-1})$.

The Applicable Decrimped Length (L - Jaw Adjustment + $e_{crimp\ take\text{-}up}$) represents the portion of the staple between the jaws, and is required to correctly convert the Extension to fractional Strain.

The cumulative sum of these small areas, CA_i, represents the product of the Applicable Decrimped Length (mm) and the Specific Work done on the staple ($N.ktex^{-1}$) up to clock tick ***i***.

$CA_i = CA_{i-1} + DA_i$.

The Specific Work of Rupture ($N.ktex^{-1}$) = $CA_{i=n}$ / Applicable Decrimped Length

One Clock Tick Analysis

The One Clock Tick method of modelling calculates the slope of the Strength vs Extension curve over one clock tick (i.e. from ***i*** - 1 to ***i***). The force measurements were stable enough to permit this approach.

The slope ($N.ktex^{-1}.mm^{-1}$) at clock tick ***i*** is $Slope1CT_i$, where

$Slope1CT_i = (S_i - S_{i-1}) / (e_i - e_{i-1})$

This is evaluated in order to assess the maximum slope after the crimp take-up and prior to the peak (MaxSlope1CT). That is deemed to occur at Extension e_i such that clock tick ***i*** corresponds to the earliest occurrence of the maximum slope. For this value of ***i***, the extension at which an extrapolation of this slope intercepts the S = 0 axis is e_{CU1CT}, where

$e_{CU1CT} = e_{i-1} - S_{i-1}$ / MaxSlope1CT

The Extension e_{CU1CT} is the Extension of Crimp Take-up according to the One Clock Tick Analysis.

The product of Specific Work of Crimp Take-up (N/ktex) and the Applicable Decrimped Length (mm) under this regime is represented by $CA_{i-1} - ½ (e_{i-1} - e_{CU1CT}) S_{i-1}$.

Initial Modulus = MaxSlope1CT x (L - Jaw Adjustment + e_{CU1CT})

Decrimped Staple Length = (L - Jaw Adjustment + e_{CU1CT}) x L / (L - Jaw Adjustment)

Work of Crimp Take-Up as % of Total Work = 100% (CA_{i-1} - ½ (e_{i-1} - e_{CU1CT}) S_{i-1}) / $CA_{i=n}$, where ***i*** is the clock tick corresponding to the earliest occurrence of the maximum slope.

The "Threshold Strength (Right Tail) Analysis" showed how the product of the Specific Work to reach the threshold and the Applicable Decrimped Length ($N.ktex^{-1}.mm^{-1}$) could be calculated. The product of the Specific Work of Crimp Take-up and the Applicable Decrimped Length ($N.ktex^{-1}.mm^{-1}$) can be subtracted from that term. Division by the Applicable Decrimped Length, (L - Jaw Adjustment + e_{CU1CT}), will produce a Specific Work of Rupture ($N.ktex^{-1}$), whereas division by ($e_{threshold=4}$ - e_{CU1CT}) produces Average Strength ($N.ktex^{-1}$).

The Staple Strength calculations use a Linear Density value based upon the unextended Staple Length. Hence this is a hybrid basis for a Specific Work of Rupture calculation.

The Point of Apparent Yield is deemed to exist beyond the occurrence of the interval of maximum slope, when the slope of the Strength vs Extension curve first drops below an arbitrary 75% of the maximum slope, i.e. $i > i_{max\ slope}$ and (S_i - S_{i-1})/(e_i - e_{i-i}) < 0.75 (S_{i-1} - S_{i-2})/(e_{i-1} - e_{i-2}). Having identified ***i***, the Strength at the Point of Apparent Yield is S_i.

Two Clock Tick Analysis

This method of modelling calculates the slope of the Strength vs Extension curve over two clock ticks (i.e. from ***i*** - 2 to ***i***). It is otherwise analogous to the One Clock Tick Analysis.

95 % Energy Analysis

This approach defines an arbitrary central domain in which Work of interest (i.e. breaking) occurs. The left limit of that domain was assumed to represent the right boundary of the Work of Crimp Take-up*.

The 95% Energy Analysis is an alternative to the Clock-Tick Analyses that makes no use of the slope of the curve.

The Extension at which 2.5 % of the Work of Rupture has been exerted, $e_{.025}$, can be calculated using a linear interpolation. The method is to find ***i*** such that $CA_{i-1} < 0.025\ CA_{i=n} \leq CA_i$. At that point $e_{.025} = e_{i-1} + (e_i - e_{i-1}) (0.025\ CA_{i=n} - CA_{i-1}) / (CA_i - CA_{i-1})$.

A similar technique was used for determining the Extension at which 97.5 % of the Work of Rupture had been exerted ($e_{.975}$). The method was to find ***i*** such that $CA_{i-1} < 0.975\ CA_{i=n} \leq CA_i$. At that point $e_{.975} = e_{i-1} + (e_i - e_{i-1}) (0.975\ CA_{i=n} - CA_{i-1}) / (CA_i - CA_{i-1})$.

The 95% Energy Analysis provided an estimate of Decrimped Staple Length with an arbitrary basis. The assumption must be made that the crimp is taken up at $e_{.025}$. Given this, and recalling the assumption that the length held in the jaws (the Jaw Adjustment) is 20 mm, under this model Decrimped Staple Length = (L - Jaw Adjustment + $e_{.025}$) x L / (L - Jaw Adjustment).

With this analysis, the Specific Work of Rupture ($N.ktex^{-1}$) = 0.95 $CA_{i=n}$ / (L - Jaw Adjustment + $e_{.025}$). The denominator is the Applicable Decrimped Length**.

The Average Strength ($N.ktex^{-1}$) = 0.95 $CA_{i=n}$ / ($e_{.975}$ - $e_{.025}$).

Threshold Strength (Right Tail) Analysis

4 $N.ktex^{-1}$ was arbitrarily chosen as a threshold for the right hand tail of the Strength vs Extension curve. Beyond this figure, the measured work done extending the staple was assumed to be due to either signal noise or fibres sliding over each other, and could be ignored.

The technique to find the extension $e_{threshold=4}$ at which this occurs was to identify ***i*** at some clock tick after the Peak Strength had occurred such that $S_i < 4\ N.ktex^{-1} \leq S_{i-1}$.

Having identified the appropriate clock tick, $e_{threshold=4} = e_i - (e_i - e_{i-1}) (4 - S_i) / (S_{i-1} - S_i)$.

The product of the Specific Work exerted to reach this threshold ($N.ktex^{-1}$) and the Applicable Decrimped Length (mm) is $CA_{threshold=4}$, where

$CA_{threshold=4} = CA_i + DA_i (e_{threshold=4} - e_{i-1}) / (e_i - e_{i-1})$.